Interpolating Moving Least-Squares-Based Meshless Time-Domain Method for Stable Simulation of Electromagnetic Wave Propagation in Complex Shaped Domain

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To stabilize the electromagnetic wave propagation simulations using a meshless time-domain method (MTDM) in complex shaped domains, the new MTDM embedding the shape functions generated by the interpolating moving least-squares method (IMLS) has been developed. Numerical experiments show that the new MTDM can employ a relatively large time step for the simulations in comparison with that of the conventional one. In addition, the parameters for generating shape functions of new MTDM can be chosen more flexibly than those of the conventional one.

***Index Terms*—Electromagnetic propagation, Waveguide bends, Maxwell equations, Time-domain analysis**

I. Introduction

MESHLESS methods have been applied to numerical simulations in a lot of fields, including the electromagnetics, and have produced many attractive results [1], [2], [3]. In particular, a meshless method based on the radial point interpolation method (RPIM) [4] has been applied to electromagnetic wave propagation simulations [1]. We refer to this as a meshless time-domain method (MTDM). MTDM does not require the rectangle meshes that are usually required in the finite-difference time-domain method (FDTD). Hence, the node alignment of MTDM is more flexible than that of FDTD, and MTDM has been applied to the simulations in complex shaped domains [3]. However, to execute the simulations in the complex shaped domains stably, it is indispensable that some parameters for generating RPIM-based shape functions are appropriately set. For this reason, it is sometimes difficult that MTDM with RPIM-based shape functions is simply applied to the simulations in the complex shaped domains.

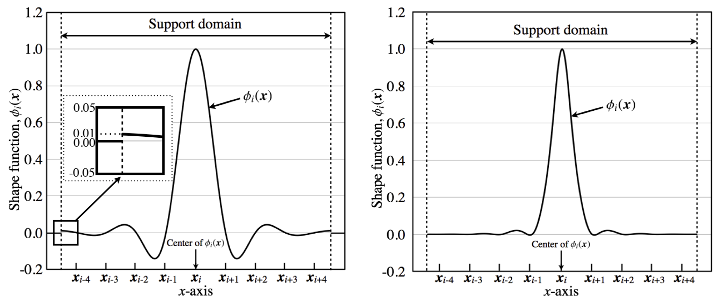
On the other hand, the shape functions for MTDM must satisfy the Kronecker delta function property [4]. Since the RPIM-based shape functions satisfy the property, MTDM has employed the ones. Note that shape functions generated by the interpolating moving least-squares method (IMLS) [5] also satisfy the property. By using the IMLS-based shape functions, the simulations by MTDM may be stable.

The purpose of the present study is to stabilize the electromagnetic wave propagation simulations using MTDM in complex shaped domains. To this end, MTDM embedding the IMLS-based shape functions is developed, and the performance of the one is investigated.

II. Meshless Time-Domain Method

To simulate electromagnetic wave propagation, we consider the Maxwell equations in case of the 2-D TM mode. The discretized forms of these equations by MTDM are as follows [1]:

$$E_{z,i}^{n} = \frac{\left(\dfrac{\varepsilon}{\Delta t} - \dfrac{\sigma}{2}\right) E_{z,i}^{n-1} + \displaystyle\sum_{j=1}^{N^H} \left(H_{y,j}^{n-\frac{1}{2}} \frac{\partial \phi_{j,i}^{H}}{\partial x} - H_{x,j}^{n-\frac{1}{2}} \frac{\partial \phi_{j,i}^{H}}{\partial y} \right)}{\dfrac{\varepsilon}{\Delta t} + \dfrac{\sigma}{2}}, \quad (1)$$



(a) (b)

Fig. 1. Shape functions generated by (a) RPIM and (b) IMLS.

$$H_{x,j}^{n+\frac{1}{2}} = H_{x,j}^{n-\frac{1}{2}} - \frac{\Delta t}{\mu} \sum_{i=1}^{N^E} E_{z,i}^{n} \frac{\partial \phi_{i,j}^{E}}{\partial y}, \quad (2)$$

$$H_{y,j}^{n+\frac{1}{2}} = H_{y,j}^{n-\frac{1}{2}} + \frac{\Delta t}{\mu} \sum_{i=1}^{N^E} E_{z,i}^{n} \frac{\partial \phi_{i,j}^{E}}{\partial x}, \quad (3)$$

where n is the number of iterations, Δt is a time step, E_z is the z component of the electric field $\boldsymbol{E}$, and H_x and H_y are the x and y components of the magnetic field $\boldsymbol{H}$, respectively. In addition, $\phi_i^E(\boldsymbol{x})$ and $\phi_j^H(\boldsymbol{x})$ denote the shape functions corresponding to $\boldsymbol{x}_i^E$ and $\boldsymbol{x}_j^H$, respectively. Here, $\boldsymbol{x}_i^E (i = 1, 2, \ldots, N^E)$ and $\boldsymbol{x}_j^H (j = 1, 2, \ldots, N^H)$ are nodes for discretizing $\boldsymbol{E}$ and $\boldsymbol{H}$, respectively. Note that $E_{z,i}^n \equiv E_z^n(\boldsymbol{x}_i^E)$, $H_{x,j}^{n+1/2} \equiv H_x^{n+1/2}(\boldsymbol{x}_j^H)$, $H_{y,j}^{n+1/2} \equiv H_y^{n+1/2}(\boldsymbol{x}_j^H)$, $\phi_{i,j}^E \equiv \phi_i^E(\boldsymbol{x}_j^H)$ and $\phi_{j,i}^H \equiv \phi_j^H(\boldsymbol{x}_i^E)$.

III. Shape Functions for MTDM

To derive (1), (2), and (3), it is assumed that the shape functions satisfy a property [4],

$$\phi_i(\boldsymbol{x}_j) = \delta_{ij}, \quad (4)$$

where δ_{ij} is the Kronecker delta. Note that, in this section, we abbreviate the superscripts, E and H, in shape functions.

In MTDM, RPIM-based shape functions [4] have been used, since the shape functions satisfy (4). However, electromagnetic wave propagation simulations using MTDM may be sometimes unstable, especially for complex shaped domains. Here, an RPIM-based shape function for 1-D case is shown as Fig. 1(a).

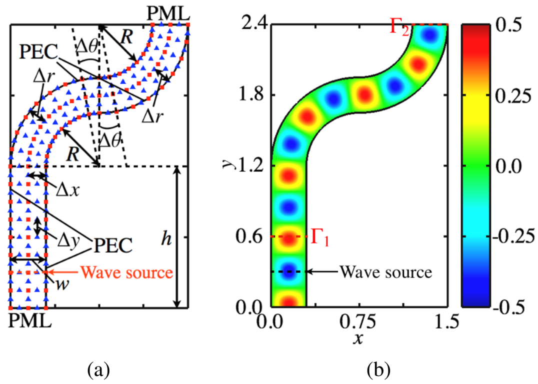


Fig. 2. (a) Schematic of a waveguide bend for experiments. Node alignment of $\boldsymbol{x}^E$ and that of $\boldsymbol{x}^H$ are represented as red quadrilaterals and blue triangles, respectively. Here, $w = 0.3$(m), $h = 1.2$(m), $R = 0.45$(m). (b) Distribution of the electric field E_z obtained by IMLS-MTDM.

TABLE I
Dependence of stability of simulations on α for $N = 302645$ and $N_s \in [6, 12]$.

α	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
RPIM-MTDM	✓	✓	✓	✓	N/A	N/A	N/A	N/A	N/A
IMLS-MTDM	✓	✓	✓	✓	✓	✓	✓	✓	N/A

We see from this figure that $\phi_i(\boldsymbol{x})$ is discontinuous on the boundaries of support domain. We consider that the discontinuity of $\phi_i(\boldsymbol{x})$ may cause the unstable simulations.

Recently, we found another choice of shape functions that satisfy (4). The shape functions are generated by the interpolating moving least-squares (IMLS) [5]. An IMLS-based shape function for 1-D case is shown as Fig. 1(b). We see from this figure that $\phi_i(\boldsymbol{x})$ smoothly becomes zero on the boundaries of support domain. For this reason, we consider that the simulations using MTDM may be stable by using IMLS-based shape functions.

IV. Numerical Experiments

In this section, numerical experiments are conducted to investigate the performance of MTDM with IMLS-based shape functions for 2-D electromagnetic wave propagation simulations in a waveguide bend illustrated in Fig. 2(a). In this figure, parameters are fixed at $w = 0.3$(m), $h = 1.2$(m), and $R = 0.45$(m). In addition, we assume that the wave source is a sine wave whose amplitude, frequency and speed are 1.0 (V/m), 1.0×10^9 (Hz) and 299792458 (m/s), respectively.

In the following, RPIM-MTDM and IMLS-MTDM denote MTDMs with RPIM- and IMLS-based shape functions, respectively. Throughout this section, we employ square-shaped support domains to generate shape functions either by RPIM or by IMLS. The size of each support domain is determined so that the number N_s of nodes contained in the support domain is $N_s \in [N_{\min}, N_{\max}]$, where $N_{\min}$ and $N_{\max}$ are parameters.

First we investigate the stability of electromagnetic wave propagation simulations by RPIM- and IMLS-MTDMs. Here, we show a stable condition for MTDM as $v\Delta t \le \min|\boldsymbol{x}_i^H - \boldsymbol{x}_j^H|$ $(i, j = 1, 2, \dots, N^H; i \ne j)$ [3], where v denotes wave speed. To satisfy this equation, we set $\Delta t = \alpha \min|\boldsymbol{x}_i^H - \boldsymbol{x}_j^H|/v$ $(i, j = 1, 2, \dots, N^H; i \ne j)$, where $0 < \alpha < 1$. TABLE I shows the dependence of the stability of simulations by RPIM- and IMLS-MTDMs on the value α for $N_{\min} = 6$, $N_{\max} = 12$ and $N = 302645$, where $N = N^E + N^H$. For the case where the simulation can be executed until $t = 50000\Delta t$, "✓" is written in this table. We see from this table that Δt of IMLS-MTDM can be set as relatively large values in comparison with that of RPIM-MTDM. Here, for $\Delta x = \Delta y = \Delta r = 1/300$(m), $\Delta\theta = \pi/480$, $t = 50000\Delta t$, and $\alpha = 0.85$, the distribution of E_z obtained by IMLS-MTDM is shown as Fig. 2(b). We see from this figure that E_z is smoothly distributed in the waveguide.

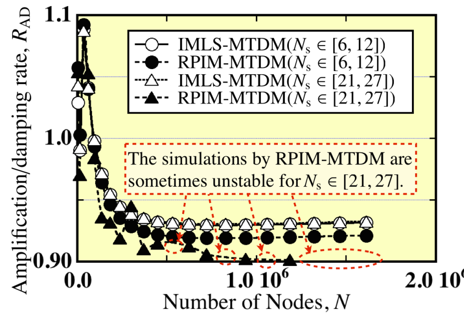


Fig. 3. Dependence of the amplification/damping rate R_{AD} on the number N of nodes. Here, N_s is the number of nodes contained in the support domain.

Next, we investigate the convergence of an amplification/damping rate R_{AD} defined as

$$R_{\mathrm{AD}} \equiv \left\langle \int_{\Gamma_2} |\boldsymbol{E} \times \boldsymbol{H}| \ \mathrm{d}\ell \right\rangle_t \Bigg/ \left\langle \int_{\Gamma_1} |\boldsymbol{E} \times \boldsymbol{H}| \ \mathrm{d}\ell \right\rangle_t . \qquad (5)$$

In (5), to calculate R_{AD}, y-coordinate of Γ_1 and that of Γ_2 are set as 0.6 and 2.4, respectively (see Fig. 2(b)). For $\alpha = 0.5$, the dependence of R_{AD} on the number N of nodes is shown in Fig. 3. We see from this figure that R_{AD} of IMLS-MTDM converges about 0.93 for both cases of $N_s \in [6, 12]$ and $N_s \in [21, 27]$. In addition, R_{AD} of RPIM-MTDM converges about 0.92 for $N_s \in [6, 12]$. In these cases, the electromagnetic wave is slightly damped, since $R_{\mathrm{AD}} < 1$ at convergence. We consider that this damping is caused by wave reflections in the waveguide bend. Note that R_{AD} of RPIM-MTDM for $N_s \in [21, 27]$ is sometimes not plotted in Fig. 3. This is because the simulations by RPIM-MTDM are unstable, i.e., we can not calculate R_{AD} in some cases illustrated as dotted ellipses in this figure. From these results, we consider that a flexibility for setting parameters of $N_{\min}$ and $N_{\max}$ in IMLS-MTDM is more superior than that in RPIM-MTDM.

From the above results, we conclude that the electromagnetic wave propagation simulations by IMLS-MTDM is more stable than those by RPIM-MTDM.

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